Thinking Skills and Technology

**Question 1** Solve the inequality \( f(x, y, z) = 2x - 3y - 2z - 11 + 5\sqrt{x^2 + y^2 + 11 + 4z^2} \leq 0 \). Use CAS, if it is convenient.

**Question 2** Let \( f(x, y, a) = 1 - \sqrt{x^2 - 2ax + y^2 - 4y + 5} \). The expression gets its maximum value only for a single pair \((x, y)\). Find the range of the parameter \( a \in [-1,1] \).

**Question 3** Let \( N \) be the Nagel point of triangle \( ABC \). Let \( T_A, T_B, \) and \( T_C \) be the extouch points at which the \( A \)-excircle meets line \( BC \), the \( B \)-excircle meets line \( CA \), and \( C \)-excircle meets line \( AB \), respectively. Let \( P_A \) be a point on \( AT_A \) such that \( AP_A = NT_A \). Let \( P_B \) be a point on \( BT_B \) such that \( BP_B = NT_B \). Let \( P_C \) be a point on \( CT_C \) such that \( CP_C = NT_C \). Find the incenter of \( P_A P_B P_C \) triangle.

**Note:** "Nagel point" from Wikipedia:

In geometry, the Nagel point is a point associated with any triangle. Given a triangle \( ABC \), let \( TA, TB, \) and \( TC \) be the extouch points in which the \( A \)-excircle meets line \( BC \), the \( B \)-excircle meets line \( CA \), and \( C \)-excircle meets line \( AB \), respectively. The lines \( ATA, BTB, CTC \) concur in the Nagel point \( N \) of triangle \( ABC \). The Nagel point is named after Christian Heinrich von Nagel, a nineteenth century German mathematician, who wrote about it in 1836.