Problem 1

Suppose that there are \( n \) people in a room. What is the exact probability that there are exactly two people \((A, B)\) such that \( A \) and \( B \) have the same birthday and no others share a birthday? Note that this is a variation of the famous birthday paradox where the question is to determine the probability that at least two of \( n \) people have the same birthday. Determine the probability for \( n=5 \) as an exact fraction. Give a decimal approximation for \( n=30 \). Then give a formula for the general case for any \( n>2 \). Challenge: Extend the problem for two distinct pairs \((A, B), (C, D)\) such that \( A, B \) share a birthday, and \( C, D \) share a different birthday and everyone else has a distinct birthday.

Problem 2

Mr. Rich has purchased a piece of diamond in a right pyramid shape; the diamond has a square base ABCD of each side 16mm. Each sloping edge is 24mm long. Mr. Rich plans to embed this piece of diamond into the pendant of his wife’s necklace so he needs to know the angle between the faces VAB and VBC. Can you help him? (The following picture is from “Surface Area and Volume of Pyramids” at http://www.ck12.org/geometry/Surface-Area-and-Volume-of-Pyramids/lesson/Pyramids/