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Adapting Mathematics Education to the Needs of ICT

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Abstract

By looking the relationship between technology and mathematics education from five perspectives, the article suggests that instead of speaking about 'implementing modern technology into classroom' it might be more appropriate to speak about 'adapting mathematics teaching to the needs of information technology in modern society'. This means emphasizing more the making of informal than formal mathematics within the framework of eight main activities and motives, which have proved to be sustainable in the history of human thinking processes and making of mathematics. Concerning the paradigm shift of school teaching, the article discusses the dilemma between systematic models and minimalist instruction.

Introduction

Technology-based (or ICT-based) mathematics education has expanded to include the following solutions, many of those being used via networks or in local computers, including modern calculators and communicators (Haapasalo & Silfverberg 2007):

- computer algebra systems (CAS), dynamical geometry (DGS), and dynamical statistics (DSS);
- spreadsheets, drawing programs, and other versatile tools for mathematical modelling;
- online databases of available software, instruction, research, statistics, history, etc.;
- online communication in all of its synchronous and asynchronous forms;
- new kinds of environments to read, write and publish, including tools for support;
- tools for utilizing of the world-wide web: search engines, etc.;
- online experiments and simulations in diverse forms of digital educational content
- online libraries containing books, learning objects, other teaching materials, digital portfolios, etc.
- learning management systems (LMS), which are used to manage students and course materials;
- virtual worlds in the form of three-dimensional immersive environments offering, for example, shared exhibitions or other forms of collaborative functionality.

These opportunities together with a changed conception of knowledge and learning could lead to a paradigm shift: learning of mathematics is more distributive (independent of time, place and formal modes), constructivist (learning community centred) and technologically enhanced. Even though students use technological applications in more informal way, as on their free time, for too many curriculum designers and teachers, technology-supported learning environments appear as "interactive e-textbooks", based on objectivist-behaviorist tradition to learn basic facts and skills[1].

Kadijevich (2004) points out four areas that have been neglected in research in mathematics education:

[1] Haapasalo & Silfverberg (2007) describe in detail, how the Finnish school curriculum neglects ICT opportunities even though students' top scores in PISA studies are not due to classroom activities. They also describe quite similar situation in other countries. In UK at Key Stages 3-4, for example, more than ¾ of the students never or hardly ever used ICT during mathematics lessons.

promoting the human face of mathematics; relating procedural and conceptual mathematical knowledge; utilizing mathematical modelling in a humanistic, technology-supported way; and promoting technology-based learning through applications and modelling, multimedia design, and on-line collaboration. Haapasalo and Siekkinen (2005) find support for the following hypotheses: Technology can enhance learning skills (metacognitions) among teachers and students; it is reasonable to utilize minimalist instruction especially when technology concerns; technology can shift learning from the classroom into free time; technology-based learning can benefit from the 'learning by design' principle; and that the most appropriate way to implement technology in teacher training is to use it as a solid part of knowledge structure and of student pedagogical thinking. Based on these two studies and the ongoing researches within my doctoral students I decided to write this article within the following categories concerning what modern technology can maintain and promote: (1) Links between conceptual and procedural knowledge, (2) Metacognitions and problem-solving skills, (3) Sustainable components of mathematics making, (4) Interplay between systematic approaches and minimalist instruction, and (5) Learning by design. By using examples of empirical studies I will try to emphasize that these aspects might very often be related to technology in more natural way outside the classroom than within insitutional teaching.

Links between conceptual and procedural knowledge

There is a basic conflict between conceptual and procedural knowledge[2]: how much students should understand before they are able to do, and vice versa. Concerning ICT-based learning, the first challenge arises from the structure of the topic to be learned, whereas the other is caused by the instructional variables required for technology use. For too many students, one of the basic difficulties for the learning of mathematics is that very often entities appear as well as objects as processes. Kadijevich (2006) stresses the respect of the following two requirements: (1) when utilize mathematics, don't forget available tool(s); when make use of tool, don't forget the underlying mathematics; and (2) to solve the assigned task, use, whenever possible, a process approach as well as an object approach, working with different representations (algebraic and graphical, for example). These demands can be realized if the teacher has fundamental know-how of the relation between conceptual and procedural knowledge. According to Rittle-Johnson and Koedinger (2004), the two knowledge types seem to develop iteratively, where a change of problem representation influences their relation. Such a development was assumed in the pedagogical model of the *MODEM*-project[3]. This model for the interplay between the two knowledge types makes use of spontaneous procedural knowledge as well as the simultaneous activation of conceptual and procedural knowledge (see Figure 1).

[2] I adopt the following characterizations of Haapasalo and Kadijevich (2000):
• *Procedural knowledge* denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
• *Conceptual knowledge* denotes knowledge of particular networks and a skilful "drive" along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

[3] See Haapasalo (2003) and http://www.joensuu.fi/lenni/modemfin.html. To see the systematic approach of the MODEM-framework, the CAL–software is freely downloadable at http://www.joensuu.fi/lenni/programs.html.

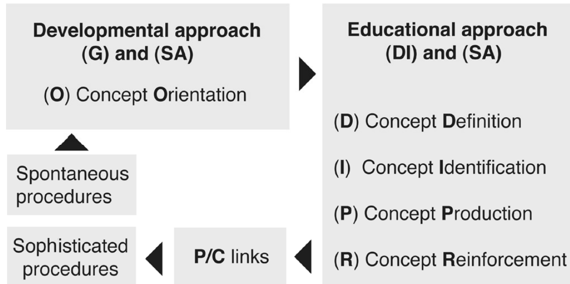

Figure 1. MODEM framework as sophisticated interplay between developmental and educational approach

The logical relation between the two knowledge types in the *developmental approach* is based on a *genetic view* (i.e. procedural knowledge is necessary for the conceptual) or a *simultaneous activation view* (i.e. procedural knowledge is necessary and sufficient for conceptual knowledge)[4]. Nevertheless, it seems appropriate to claim that the goal of any education should be to invest in conceptual knowledge from the very beginning. If so, the logical basis of this *educational approach* is the *dynamic interaction view* (i.e. conceptual knowledge is necessary for the procedural), or again the simultaneous activation view. Such a simultaneous view assumes that the learner has opportunities to activate simultaneously conceptual and procedural features of the current topic. By "activating" I mean certain mental or concrete manipulations of the representations of each knowledge type. The left-hand side of Figure 2 illustrates how the simultaneous activation principle is utilized in problem posing: "Move the end points of the lines with the mouse and see how k_1 and k_2 change". Figures in the middle and on the right illustrate students' solutions to be represented later [5].

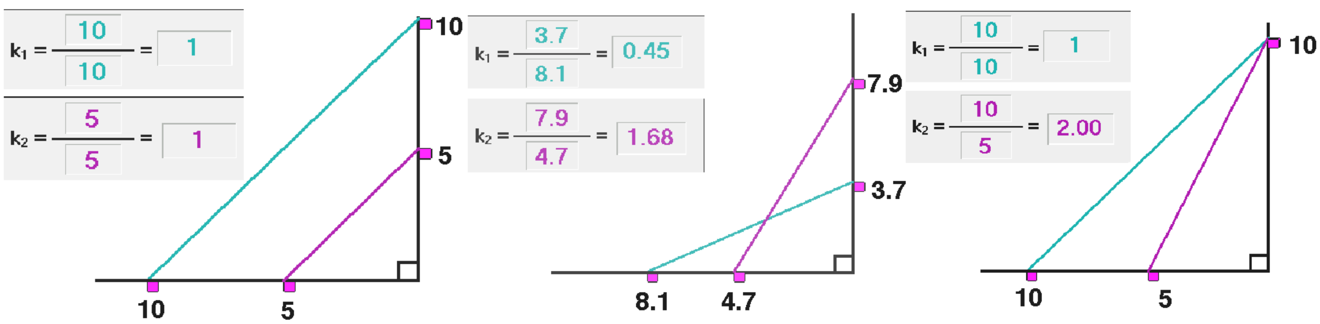

Figure 2. Utilizing the SA method in technology-based learning environment

Figure 3 below shows how today even small pocket computers can be utilized for the same purpose. One such computer is *ClassPad* made by Casio (see http://www.classpad.org). The drag-and drop technology allows the student to manipulate mathematical objects between two windows, illustrating two different forms of mathematical representation. In many cases this means forming links between conceptual and procedural knowledge. The *Geometry Link* –operation carrries student's manipulation between the two windows without any drag-and-drop maneuvre. We will see in student's portfolio that those properties are really utilized in sophisticated way without any tutoring from teacher's side.

[4] Four views can be found in the literature on the logical relationship between conceptual and procedural knowledge (cf. Haapasalo & Kadijevich 2000). The two approaches here are based on these views.

[5] I skip representing the model of Figure 1 in detail and refer to footnote #3 to see how to move from the concrete slope to the abstract concept gradient, and how the mathematical concept building can be scaffold by utilizing the dynamic interaction.

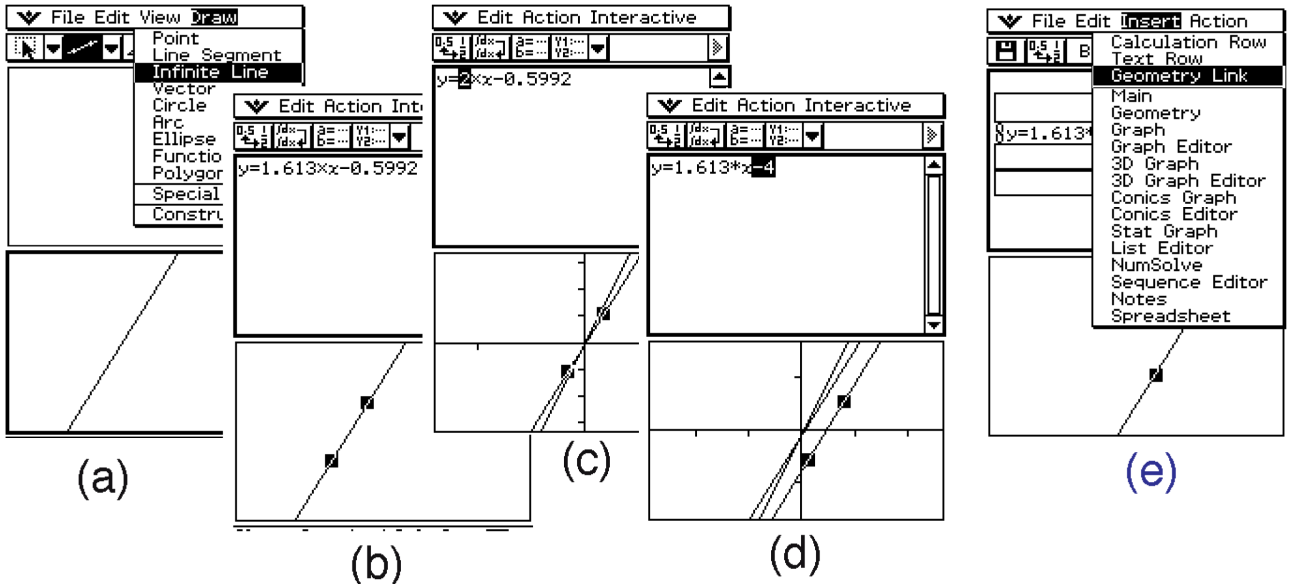

Figure 3. Utilizing SA-method through drad-and-drop technology (a-d) or Geometric Link (e).

Relating different representations can not only support the development of conceptual knowledge (cf. Papert 1987), but also relate procedural and conceptual knowledge (cf. Haapasalo 2003, Kadijevich & Haapasalo 2001). Because of that, although summarized in a somewhat simplified way, it can be said that to coordinate the process and object features of mathematical knowledge, multiple forms of representation are to be utilized and connected, especially with the aid of modern technological tools. The use of these tools should not reinforce a strictly hierarchical nature of mathematical knowledge but rather promote its quality of a flexible network (Kadijevich 2006).

Metacognitions and problem-solving skills

To avoid the fact that this dimension is too general and perhaps therefore stale and flat, I would like to invite the reader to think that students very often use modern technology in very sophisticated way outside the classroom. By applying minimalism (to be explained later) Eronen & Haapasalo (2006) gave students at 8th class opportunity to study voluntarily 9th class mathematics with ClassPad during their summer holiday. This totally new tool was shortly represented to them just few days before their summer holiday. The only duty was to write a portfolio if they worked with the tool. The following example is taken from the portfolio of a quite average student. The letters (a)-(d) refer to Figure 3.

Example of student's 6th session on 15th of July 2005. Time 00.27

- I draw a line (cf. a). When drag-dropping, the equation of the line is y= **1.613**x-0.5992 (b).
- By changing the equation to y=**2**x-0.5992 the angle between the line and y-axis is getting smaller (c).
- By changing the equation to y=**1**x-0.5992, the angle between the line and y-axis is getting bigger.
- I change the equation to y= 1.613x**-0.4**. I don't see any changes.
- I change the equation to y= 1.613x**-4**, the line moves to the same direction away from origin (d).
- When changing the equation to y= 1.613x**+4,** the line moves in the same way, but to another direction on x-axis with equal distance from the origin.
- I will continue in the morning. Time is now 01.42. I worked 1 h 15 min.

However, Figure 2 shows that school teaching seems to contaminate those skills even though the task would be tailored to concentrate solely on mathematical and pedagogical aspects (i.e identification task within MODEM –framework) without any features of the technical tool. The figure in the middle illustrates how students change all possible components but simply produce a flood of data. This data overload prevents them recognizing the essential aspects. To handle as an expert learner (on the right) - as they perhaps would do on their free time - they would see the relevant attributes by dragging the end point of just one segment with the mouse. I have met one or two such learners in more than 15 years of tutoring with this software among hundreds, maybe thousands, of students and teachers observed in schools and universities. This can be interpreted as showing that these institutions do not promote or maintain general ideas of Polya's (1979) checklist, for example.

As an example of constructivist software for distributive learning I would like to mention *Kidware* aquarium simulation program[6], which allows even young children to discover the core concepts of balanced ecosystem (how e. g. parameters of warmth and air affect the quality of water, which in turn affects the health of fish in the aquarium; see Figure 4). This kind of software contains an artificial model of a system and processes, in which conceptual and procedural knowledge is embedded to be applied in knowledge representations with a very strong relation to constructivist learning. Experiences with this program as well inside as outside the classroom are very encouraging. Haapasalo & Siekkinen (2005) report that after using these programs during two years, children's metacognitions increased, whilst entertainment- relatedness decreased. I had opportunity to gather reinforcement for these kinds of findings by observing all of my own four children during several years. Amazingly they could solve with the software complicated ecological problems without any tutoring from adult's side.

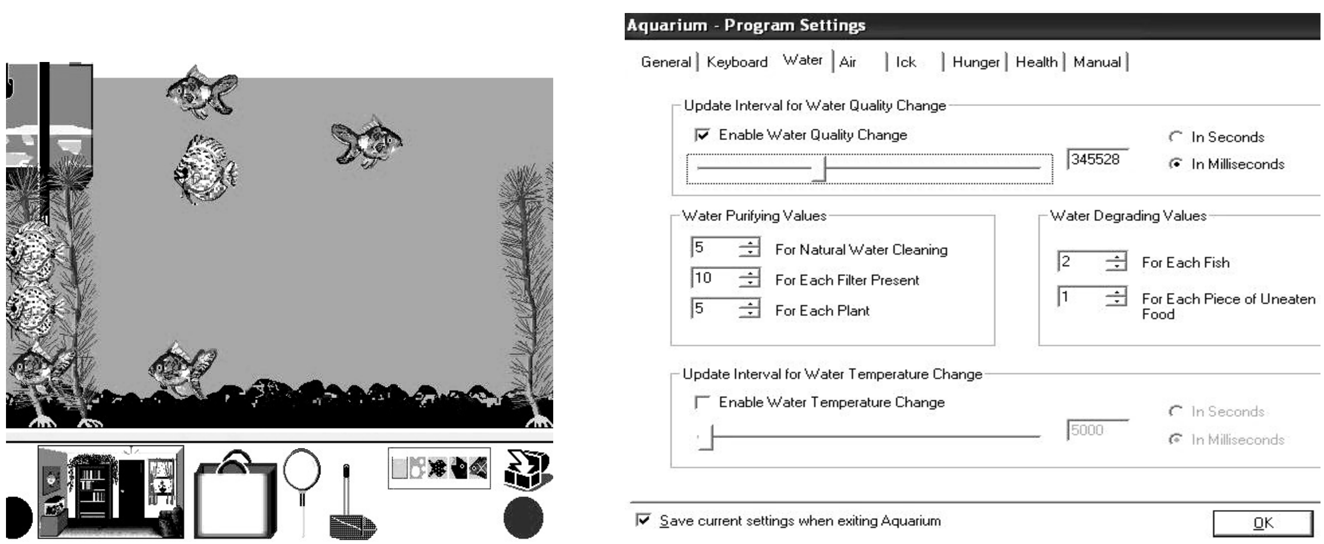

Figure 4. Aquarium simulation environment (left), and setting of conceptual and procedural controls (right).

Sustainable components of mathematics making

Zimmermann's (2003) long-term study of the history of mathematics reveals eight main motives and activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years. We (Eronen & Haapasalo 2006) took this network of activities illustrated in Figure 5 as an element in our theoretical framework for the structuring of

[6] See Mobius (2002) and http://www.kidware.com/mobius/KWbroch.htm. A Finnish animation of the aquarium simulation can be downloaded at http://www.edu.joensuu.fi/siekkinen/aquarium.wmv on the permission of Dr. Siekkinen.

learning environments and for analyzing student's cognitive and affective variables. The 'find-corner' represents heuristic activities in the sense of Pólya. Within these activities ClassPad study focuses on "changing representation" which is not only a powerful thinking tool to enhance problem solving processes but it might also promote links between procedural and conceptual knowledge.

As an example I represent affective results concerning the very same student who produced the portfolio sample above. She can be considered to be not especially motivated in learning mathematics during her studies neither on 7th nor on 8th class. The left-hand side of Figure 5 represents her view of mathematics in May (i.e. before the ClassPad work) and August (i.e. after that one), showing some a shift from 'apply' to 'find' and 'argue'. The figure in the middle shows that there is the same kind of shifting in her self-confidence. The interview revealed expressions like: *"Now I know better and see things in different light"*. However, more interesting one is student's conception of computer's role in making mathematics. Working with ClassPad has shifted the profile even more to creative direction, the biggest change being in playing. In the interview the student expressed, for example: *"In May I could not even think to play ClassPad in summer holiday. However, I noticed, that it was very capable for playing with mathematics.*" Maybe the most surprising shifting has been occurred by 'calculate' dimension. The interview gives us an explanation. The student is becoming to see the versatility of the technical tool, which decreases the relative amount of the counting belief: *"ClassPad is suitable for calculating, but if you want to learn how to calculate, you have to do something by hand"*.

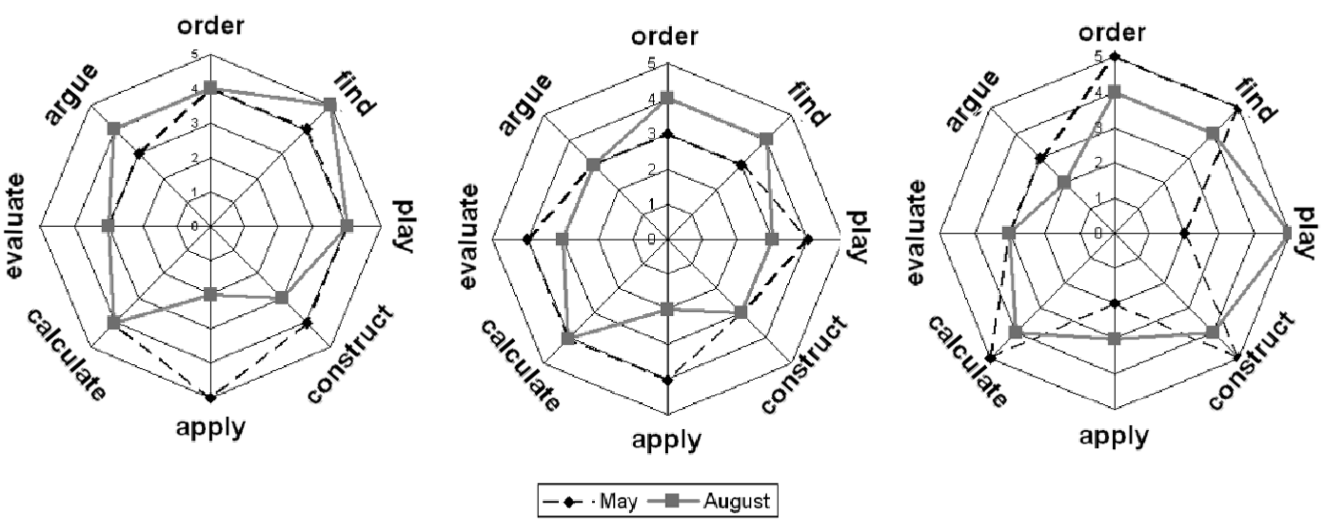

Figure 5. Student's view on mathematics before and after her ClassPad work. 'What is mathematics all about' (on the left), 'How good I am in making mathematics' (in the middle), and 'What kinds of mathematics can be made by using computer' (on the right).

Modern technology can not only promote those eight dimensions of mathematics making but it can also revitalize beautiful mathematical ideas, which have been developed by great mathematicians through centuries. Not long ago, envelopes of curves, involutes, caustics, and parallels, for example, were standard topics for freshmen. The rich concept *curve* served as an assembler of the isolated parts of mathematics: Geometry, Algebra, Trigonometry, Analytical Geometry, Calculus. As a result of moves towards generalization and rigor, especially in mathematics education, the *special curves* and most of the vital geometric spark, which has ignited so many minds in the past, have been cancelled. At school, even the study of hyperbolas and parabolas has degenerated into treating them as graphs of functions. Analysis courses at school or university maintain such an unsatisfactory view with even non-obligatory courses such as Differential Geometry using special curves for illustrative purposes only. As a result mathematics teachers are not aware of the educational potency of those curves as a fruitful field for exploration with geometric, kinematic, algebraic and other pre-calculus tools.

Our international MODEM –project[7] was an effort to develop mathematics curriculum on the university level in the guidelines emphasized above. Although the project did not produce a lot of materials, co-operation gained within the project still continues within European teacher exchange program. On the other hand, it might be proper to mention that among the huge amount of material on the Internet developed widely, there might be useful for any educator who can use search engines in sophisticated way and has proper conceptual mathematical knowledge. This opportunity was utilized by Kadijevich & Haapasalo (2004) when students designed their own hypermedia utilizing Internet.

Interplay between systematic approaches and minimalist instruction

Students very often neglect teacher's tutoring, or they feel they do not have time to learn how to use technical tools. Teachers similarly feel they do not have time to teach how these tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be accessed without first reading heavy manuals. The term *minimalist instruction*, introduced by Carroll (1990), is crucial not only for teachers but also for those who write manuals and help menus for the software. Carroll observed that learners often tend to "jump the gun". They avoid careful planning, resist detailed systems of instructional steps, tend to be subject to learning interference from similar tasks, and have difficulty recognizing, diagnosing, and recovering from their errors. I would like to pick up especially the following characteristics of minimalist instruction (cf. Lambrecht 1999):

- Specific content and outcomes cannot be pre-specified, although a core knowledge domain may be specified;
- Learning is modelled and coached for students with unscripted teacher responses;
- Learning goals are determined from authentic tasks stressing doing and exploring;
- Errors are not avoided but used for instruction;
- Learners construct multiple perspectives or solutions through discussion and collaboration;
- Learning focuses on the process of knowledge construction and development of reflexive awareness of that process;
- Criterion for success is the transfer of learning and a change in students' action potential;
- The assessment is ongoing and based on learner needs.

The features of *minimalism* include several varieties of constructivism, offering also instructional assumptions. Furthermore, accepting constructivist views of teaching and learning mathematics means emphasizing the genesis of heuristic processes and the ability of students to develop intuition and mathematical ideas. This can hardly be reached without a thorough planning of the problem to be posed and studied – inside and outside the classroom. For this, empirically tested more or less systematic pedagogical models (as MODEM described above) can be helpful. When planning a constructivist approach to the mathematical concepts under consideration, the focus is on the left-hand side of Figure 1. On the other hand, when offering students opportunities to construct links between representation forms of the concept, the focus is on the right-hand box, which describes the stages of mathematical concept building. In learning situations, however, students must have freedom to "jump the gun". They must be able to choose the problems that they want to learn within continuous self-evaluation instead of relying on express guidance from teachers. Our experiences of the ClassPad project show

[7] see http://www.joensuu.fi/lenni/modem.html; cf. also http://www.joensuu.fi/mathematics/DidMat/index.html#english and http://www.joensuu.fi/mathematics/MathDistEdu/Animations2MentalModels

(see Eronen & Haapasalo 2006) that this can be realized by organizing different kinds of task types to form a "problem buffet", for example. To go for linear function, one student team initially picked quite a complicated problem series on optimizing mobile phone costs. After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set, which happened to consist of identification tasks – the very lowest level in the systematic MODEM framework, which was on the basis of the planning of the learning environments. This example shows that sophisticated interplay between a systematic approach and minimalism can be achieved even within simple pedagogical solutions.

Learning by Design

Studies of design processes have produced useful information concerning problem solving and group dynamics, for example. Eskelinen & Haapasalo (2006) uncover how different kinds of approaches and support for reflective communication affect students' conceptions of teaching and learning, group dynamics and interest in *ICT* support. The results clearly show that design of a technology-based learning environment within an adequate constructivist theory linked to the knowledge structure offers promising respond to the main challenge of teacher education: to get students understand which are the basic components for teaching and learning. The developmental approach based on spontaneous procedural knowledge seems to be appropriate concerning as well cognitive as affective variables. To apply the educational approach to stress the importance of conceptual knowledge, educator needs a lot of sensitivity concerning cognitive and emotional variables in the learning process. Our findings give also strong evidence to the position that technology should have a strong position in teacher education programs. Our findings do not support the conception that computer skills in teacher education should be taught separately from the information structures and pedagogical thinking.

There are numerous researchers who see that *Learning by Design* is one of the most sophisticated way to implement technology even for young children, opening new productive ways to develop constructively orientated teacher education and service-in-training (e.g. Ojala et al. 1996). I would like to share the famous view of Jonassen (2000) that those who learn more from the instructional materials are their developers, not users. Therefore teachers and students should design ICT-based lessons and thus become knowledge constructors rather than knowledge users. These type of activities very often profit from minimalism (cf. Eskelinen & Haapasalo 2006; Kadijevich & Haapasalo 2004).

Closing remarks

When considering technology-based learning to reinforce and implement creative thinking, the focus has been shifted from a technology-oriented viewpoint to a humanistic view, stressing cognitive, affective and social variables involved in the learning process. Mathematics educators should be aware of the way citizens use technology in modern society and how this affects those variables. What happens in institutions should have some reasonable equivalent to what happens outside the classrooms. Maybe the most remarkable finding of our ClassPad project is that the answer to the future question "*Does the allocation of learning shift from the classroom into leisure time?*" might be affirmative and that, therefore, the role of school needs a thorough re-consideration. There are many features of minimalism in our every-day life, or to be taken into account when writing or reading manuals or learning materials. For example: taking the simplest and most straightforward way to perform a task and allowing readers, users or learners to discover alternative methods and tools of their own. This kind of learning paradigm means recalling heuristic strategies that have been successful throughout the centuries of human history. This does not, however, exclude the fact that systematic models help the teachers and educators in planning, problem posing and assessment.

The history of mathematics abounds with outstanding examples of simple, but powerful ideas for organizing the content of the curriculum in a meaningful way instead of treating the same idea in several disguised forms under the guise of “spiral curriculum”. The problem of “math dropouts” has increased now that “mathematics for all” has come into fashion as a slogan. ‘Mathematics’ is normally presented as a meaningless collection of knowledge - unrelated to the experience of the students and totally uninteresting. Sterile “logical connections” seldom lead to understanding or appreciating. This has given rise to a flourishing enterprise - empirical research - which studies and characterizes the symptoms without producing a cure. We need a new approach to the teaching of mathematics but there is little hope it will emanate from this psychological perspective. Epistemological perspectives and historical sources offer much more hope. Besides, they must not be forgotten when planning curriculum or constructivist learning environments for pupils´ productive activity. The fact that students seem to learn as well mathematical as technical skills effectively outside the classroom, forces us to ask if there is something wrong inside school as far as the question “how to learn” is concerned. By using the verb ‘adapt’ in the title of this article I wanted to emphasize that technology has reached a meta-level position in our culture. It has caused a holistic change in our “mental art”, i.e. in the way we think, plan, work and evaluate. If we accept Freudenthal’s (1991) view that mathematics is mental art, the curriculum and the culture of teaching and learning of mathematics need a thorough shift.

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