The Dynamic Geometry Software as an Effective Learning and Teaching Tool

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Abstract

This article describes how the use of dynamic geometry software has helped preservice teachers develop their abilities in three aspects: 1) challenging problem solving; 2) mathematical modeling; and 3) constructing student-centered teaching projects. The examples given indicate that for some of the challenging problems that are presented to students, it is almost impossible or very hard to manually make correct drawings. To overcome this difficulty, the use of dynamic geometry software seems to be critical, or at least very desirable. In addition, the use of the software can stimulate students' insight of problem solving and provide an easy and convincing way of verifying the solution. Moreover, students can construct accurate visual representations to model real world situations very efficiently by using transformations in dynamic geometry software. This can save time significantly so that students can concentrate on more conceptual oriented tasks. Good teaching projects that take advantage of dynamic geometry software can also effectively enhance school children's mathematics learning. The supplemental materials that accompany this paper can be found online at the following URL: http://www.radford/~scorwin/eJMT/Content/Paper/jiang.

1. Introduction

The Math Education program at New York University (NYU) offers *a Technology in Mathematics Learning and Teaching* course. The primary purpose of this course is to enable middle and high school preservice teachers to experience learning and teaching mathematics with technology. A secondary purpose is the review and enhancement of the subject area knowledge in the algebra, geometry, trigonometry, probability and statistics areas. Students entering the math education program come with diverse backgrounds and understandings, including substantial knowledge and understanding in some areas of subjects and severe deficiencies and misconceptions in others. Overcoming the difficulties and building upon strengths are essential to maximal learning and achievement of expected outcomes in the program.

This course focuses on computers as a primary technology and the most beneficial software in mathematics education. Because geometry has been a week spot in school mathematics teaching, and also because the Geometers' Sketchpad (GSP) is one of the most excellent and powerful mathematics software packages, the use of GSP in learning and teaching geometry is a major part of the course, and one of the objectives of this course is to improve pre-service teachers' mathematical reasoning and proof abilities. More specifically, the course aims at increasing pre-service teachers' geometric thinking by one van Hiele level (see [13]). According to van Hiele, students progress in geometric thinking through a taxonomy of levels, and progress from one level to the next higher level is dependent on the nature of the instruction provided to students.

Many authors (see [1], [2], [4], [6], and [12]) have written articles to show how the use of GSP has facilitated students' geometry learning, and eased the burden and increased the joy of teaching geometry. Consistent with these authors, this article will mainly describe how the Technology course at NYU has helped students develop their abilities in three aspects: 1) challenging problem solving; 2) mathematical modeling; and 3) constructing student-centered teaching projects that take advantage of the use of GSP.

2. The Use of GSP and Challenging Problem Solving

According to the NCTM Professional Standards (see [10]), in order to develop students' problem solving and mathematical reasoning abilities, the teacher should select worthwhile mathematical tasks for lessons. These tasks must engage students' intellect, develop students' mathematical understanding and skills, and stimulate students to make connections and develop a coherent framework for mathematical ideas. As the instructor of the Technology course for preservice teachers at NYU, I (the author) tried to present such worthwhile tasks, which are increasingly challenging problem solving and mathematical reasoning endeavors in the GSP environment. (For clarity, in this article, preservice teachers are referred to as "students", and secondary school students are referred to as "children".)

2.1. Promoting Visual Representation

Making a drawing (visual representation) is an important problem solving strategy, especially for geometry problems. Without a drawing (more accurately, a correct drawing), it is very hard, if not impossible, to solve a geometry problem. (A correct drawing refers to one that may or may not be exactly accurate, but close enough to the accurate construction so that it correctly represents the mathematical relationship(s) described in the related problem and therefore can help stimulate students' insight into the problem.) Sometimes, however, for a geometry problem, especially a challenging geometry problem, it is not easy to make or construct a correct drawing. The following problem is an example.

Problem: Give equilateral triangle ABC with an interior point P, such that $\overline{AP}^2 + \overline{BP}^2 = \overline{CP}^2$*, and with an exterior point Q such that* $\overline{AQ}^2 + \overline{BQ}^2 = \overline{CQ}^2$*, where points C, P, Q are on a line. Find the lengths of* $\overline{AQ}$ *and* $\overline{BQ}$ *if* $\overline{AP} = \sqrt{21}$ *and* $\overline{BP} = \sqrt{28}$ (see [8]).

When students began to make a drawing for this problem, most of their drawings were similar to what is shown in Figure 1. However, more serious thinking led students realize that their drawings were far from "correct". While exploring the solution to this problem, students tried different ways to approach it. A few students did think of rotating quadrilateral APBQ -60˚ around point B, resulting in the drawing shown in Figure 2.

Based on the given condition $\overline{AQ}^2 + \overline{BQ}^2 = \overline{CQ}^2$ and some logical reasoning, students realized that $\angle$CQ'Q should be 90˚, but visually $\angle$CQ'Q is far from a right angle. This huge inconsistency alerted students that a more accurate drawing was necessary. However, no matter how much they tried, they found it was difficult to manually make a correct drawing for this problem.

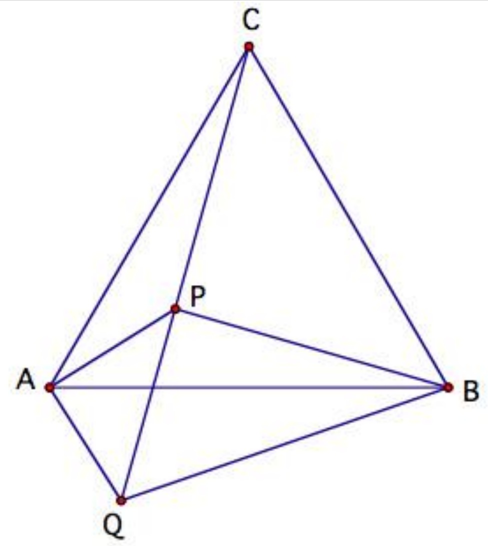


Figure 1. Incorrect drawing

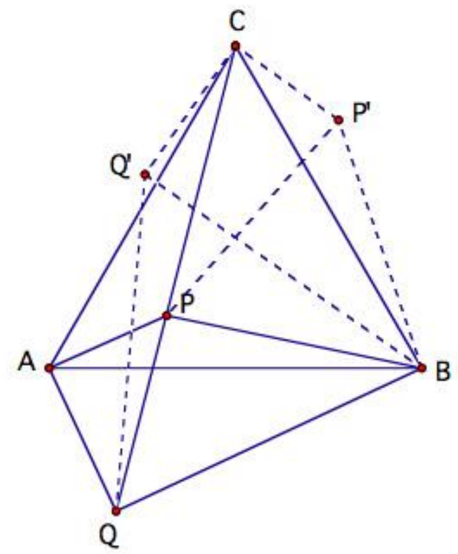


Figure 2. The drawing after a rotation

This difficulty can be easily overcome by utilizing GSP. Students could display the values of $\overline{AP}^2 + \overline{BP}^2 - \overline{CP}^2$ (v1) and $\overline{AQ}^2 + \overline{BQ}^2 - \overline{CQ}^2$ (v2) by using the "Length" and "calculate" functions in the "Measure" pull down menu. Then they could drag points P and Q respectively until both v1 and v2 were very close to 0. A resulted drawing is shown in Figure 3. It is easy to see the sharp contrast between the incorrect drawing (Figure 1) and the drawing made with GSP (Figure 3).

The contrast indicates how misleading an incorrect drawing could be. Due to intuition, however, locating Point Q (that is actually so distant away from both point C and Point P) correctly without using the dynamic geometry software would seem very difficult, if not impossible.

While most students could make a correct drawing in GSP, a few stronger students, who had gained a sound understanding of the relationships embedded in this problem, were able to even make exactly accurate drawings (constructions).

2.2. Stimulating Insights for the Solution

The most important role that GSP plays is to facilitate students' thinking. When students work on a problem, they usually would try different methods to approach the problem. The GSP dynamic movement and measurement features can help students to either confirm a method so that students know continuing to use it will most possibly achieve the solution, or provide contradicting feedback that either reveal the infeasibility of a method or stimulate a new angle of thinking.

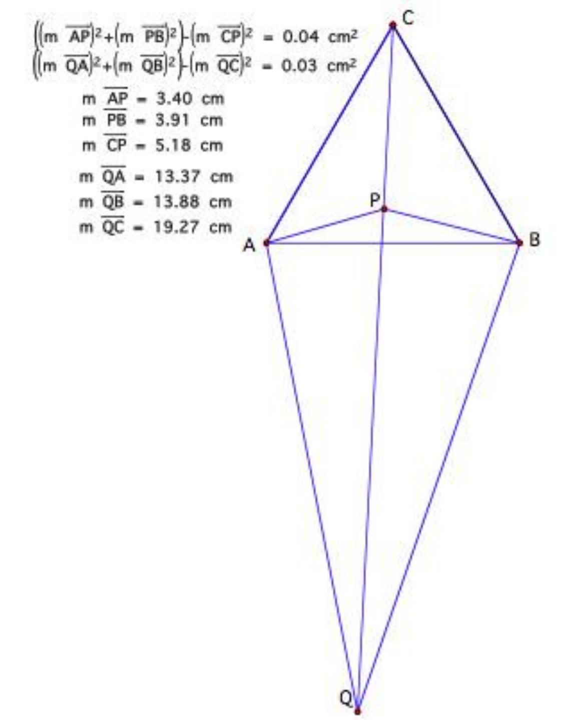


Figure 3. A correct drawing (Produced with the Geometer's Sketchpad, see [a])

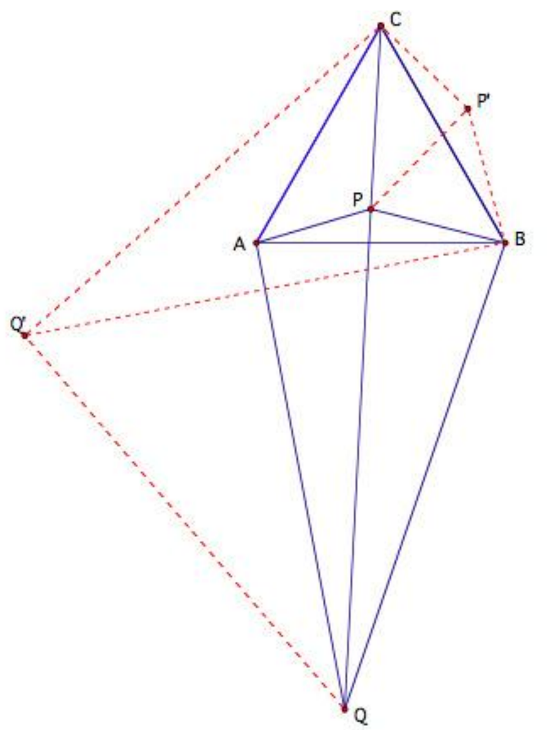
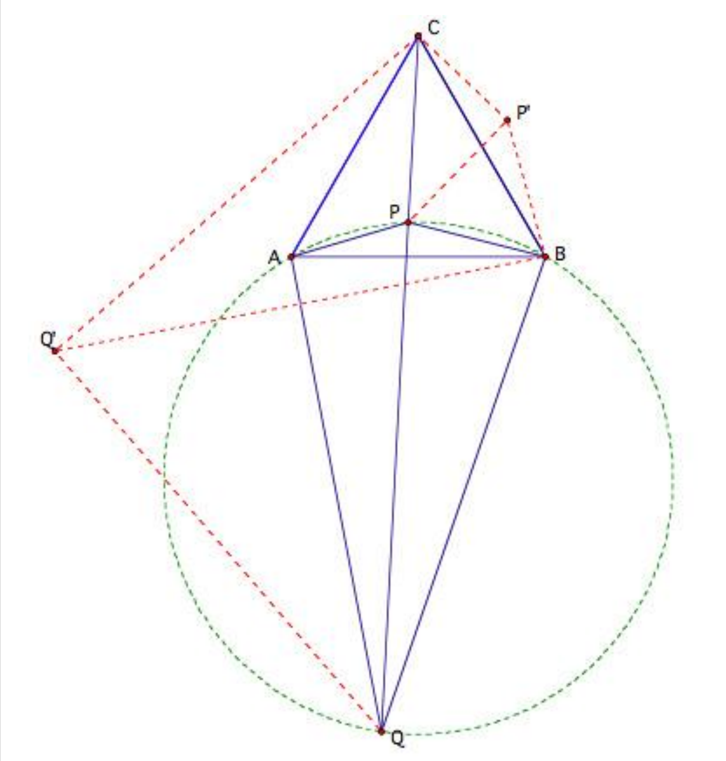


Figure 4. The drawing after a rotation

In the problem we just discussed, for example, some students thought that since points C, P, and Q are collinear and there are quite a few triangles in the figure, there might be one or more pairs of similar triangles. They first guessed that ΔCPA and ΔCAQ are similar. However, when they measured ∠CAP and ∠CQA, they found the two angles were not congruent at all. They then checked other pairs of triangles and got similar results. So they learned that trying to find some numerical relationships through using similar triangles was not feasible. So they moved on by trying different approaches. A few students thought of the numerical relationships derived from the two given conditions $\overline{AP}^2 + \overline{BP}^2 = \overline{CP}^2$ and $\overline{AQ}^2 + \overline{BQ}^2 = \overline{CQ}^2$. They reasoned that this had to do with the Pythagorean Theorem and right triangles. There were no right triangles in the current figure and so auxiliary lines should be constructed to form right triangles. They also thought of the given original equilateral ΔABC, and reasoned that 60˚ angles should be considered. Thus, they made -60˚ rotations of ΔPAB and ΔQAB around point B, and quickly found newly formed right triangles (see Figure 4). They were very excited about the discovery, and knew they were on the right track. This new approach can be found without the software, but it would be much more difficult and time consuming.

The following shows a complete solution that one of the students worked out (with minor help from the author):



Solution:

Rotate quadrilateral PAQB -60º around point B. Because ΔABC is an equilateral triangle, we have quadrilateral P'CQ'B, with ΔCP'B as the rotation image of ΔAPB and ΔCQ'B as the image of ΔAQB.

Construct segment $\overline{PP'}$. *Since* $\overline{BP'} = \overline{BP}$ *and* $\angle PBP' = 60^\circ$, *ΔBPP' is an equilateral triangle. So* $\overline{PP'} = \overline{BP} = \sqrt{28}$. *We have* $\overline{CP'} = \overline{AP} = \sqrt{21}$, *so* $\overline{CP}^2 = \overline{AP}^2 + \overline{BP}^2 = \overline{CP'}^2 + \overline{PP'}^2 = 21 + 28 = 49$. *Therefore ΔCPP' is a right triangle by the converse of the Pythagorean Theorem, and* $\angle CP'P = 90^\circ$. *Hence* $\angle CP'B = \angle CP'P + \angle PP'B = 150^\circ$. *This implies that* $\angle APB = 150^\circ$. *With a similar reasoning, we can prove that* $\angle CQ'B = 30^\circ$, *which implies that* $\angle AQB = 30^\circ$.

Therefore PAQB is a cyclic quadrilateral, as its opposite angles are supplementary.

In ΔAQP, by the Sine Theorem, $\dfrac{\overline{AQ}}{\sin(<APQ)} = \dfrac{\overline{AP}}{\sin(<AQP)}$. *But* $\angle AQP = \angle ABP$ *because they are the inscribed angles on the same arc AP, so* $\dfrac{\overline{AQ}}{\sin(<APQ)} = \dfrac{\overline{AP}}{\sin(<ABP)}$ ==> $\overline{AQ} = \dfrac{\overline{AP}}{\sin(<ABP)} \bullet \sin(\angle APQ)$.

Now let's find $\dfrac{\overline{AP}}{\sin(<ABP)}$ *and* $\sin(\angle APQ)$. *In ΔAPB, by the Cosine Theorem,* $\overline{AB}^2 = \overline{AP}^2 + \overline{BP}^2 - 2\bullet\overline{AP}\bullet\overline{BP}\bullet\cos(150^\circ) = 21 + 28 - 2\sqrt{21}\bullet\sqrt{28}\bullet(-\frac{\sqrt{3}}{2}) = 91$, *and so* $\overline{AB} = \sqrt{91}$. *By the Sine Theorem,* $\dfrac{\overline{AP}}{\sin(<ABP)} = \dfrac{\overline{AB}}{\sin(<APB)} = \dfrac{\sqrt{91}}{\sin(150^\circ)} = \sqrt{91}/(\frac{1}{2}) = 2\bullet\sqrt{91}$.

In ΔAPC, by the Cosine Theorem, $\cos(\angle APC) = \dfrac{\overline{AP}^2 + \overline{CP}^2 - \overline{AC}^2}{2\bullet\overline{AP}\bullet\overline{CP}} = \dfrac{21+49-91}{2\bullet\sqrt{21}\bullet 7} = \dfrac{-21}{14\bullet\sqrt{21}} = \dfrac{-\sqrt{21}}{14}$ ==> $\sin(\angle APC) = \sqrt{1-\cos^2(<APC)} = \dfrac{5\bullet\sqrt{7}}{14}$. *Since C, P, and Q are on a line,* $\sin(\angle APQ) = \sin(180^\circ - \angle APC) = \sin(\angle APC) = \dfrac{5\bullet\sqrt{7}}{14}$.

Therefore, $\overline{AQ} = 2 \bullet \sqrt{91} \bullet \dfrac{5 \bullet \sqrt{7}}{14} = 5 \bullet \sqrt{13}$.

Using a similar method for finding the length of $\overline{AQ}$*, or using the Cosine Theorem in* ΔAQB*, we can find that* $\overline{BQ} = 3 \bullet \sqrt{39}$.

2.3. Verifying the Solution

Another important role that GSP plays is that it can provide a good and easy way to verify the solution. In the problem that was discussed above, when the final results were found, students were not sure if they were correct answers, especially as they didn't look like "neat" or straightforward answers numerically. However, GSP dynamic measurement feature makes checking the answers quite easy and convincing. For those who constructed exactly accurate drawings, they were ready to do the verification. For others, the author provided such a drawing (construction). Students measured segments AP and AQ, calculated the ratio AQ/AP, and multiplied this ratio by $\sqrt{21}$, which is the length of segment AP. The result was approximately 18.03, and it was independent of the drawings constructed by different individuals. By a quick calculation (through GSP or a calculator), $5 \bullet \sqrt{13} \approx 18.03$. Thus, the length of segment AQ was verified. The length of segment BQ was verified in a similar way. An electronic file of an exactly accurate drawing/construction was produced with the Geometer's Sketchpad for the readers to do verifications (see [b]).

3. The Use of GSP and Mathematical Modeling

Different from general problem solving, mathematical modeling refers to processes of dealing with (usually real-world) situations that comprise information which might be incomplete, ambiguous, or undefined, with too much or too little data (see [3]). It "has to do with an open-ended problem that can be solved in a variety of ways or has many different solutions" (see [7]). One powerful feature of mathematical modeling is that students can make assumptions to simplify a complex situation so that they can start to build a model. Students will then continue to make sense of the related information, elicit and work with the embedded mathematical ideas and modify and refine their models (see [3]).

Children need to develop mathematical modeling abilities to function effectively in a world that is demanding more flexible, creative, and future-oriented mathematical thinkers and problem-solvers (see [3]). To make this happen, teachers need to have these abilities in the first place. Therefore, in the Technology course, a certain number of matheamtical modeling problems were presented to the students. The use of GSP was found to be very effective in students' exploration of the modeling problems.

Among other problems, the students explored the following real life situation:

A new restaurant will be opening in Manhattan this summer. The owner has enlisted you to help design the layout for the tables in the restaurant. Because the space is limited, she wants to maximize the number of people she is able to seat. The dining area of the restaurant is a *rectangular space 40 feet by 80 feet. Each table needs a 2.5 feet border for waiters and patrons to walk between the tables. Your task is to write her a letter and explain what kinds of and how many tables she should order.*

It can be a complicated process to figure out how many tables of different shapes and sizes should be ordered and to design the layout for these tables to maximize the number of people the restaurant is able to seat. However, as just mentioned, the students can make assumptions to simplify the situation so that they can start to build a model that they can refine later. The situation can be simplified at first by considering only one kind of table (a table of a certain shape and size, e.g., a 3 foot by 4 foot rectangular table seating 4 people or a circular table with circumference 3π feet, seating 2 people). Students can do their exploration taking advantage of technology. GSP can be used very effectively here because its transformation features allow students to quickly layout "tables". This exploration is an excellent opportunity for students to solidify their understanding of and apply proportionality when they work with the dimensions of a rectangle and the circumference, radius, and diameter of a circle. The outcome of the exploration would be a graphical representation of a uniform table layout with associated calculations. A comparison between two or more such cases will give the maximum (relative to the students' explorations) number of people that the restaurant is able to accommodate.

Figure 5 shows part of a student's modeling solution to the Restaurant problem from which we can see the GSP transformation functions were used intensively.

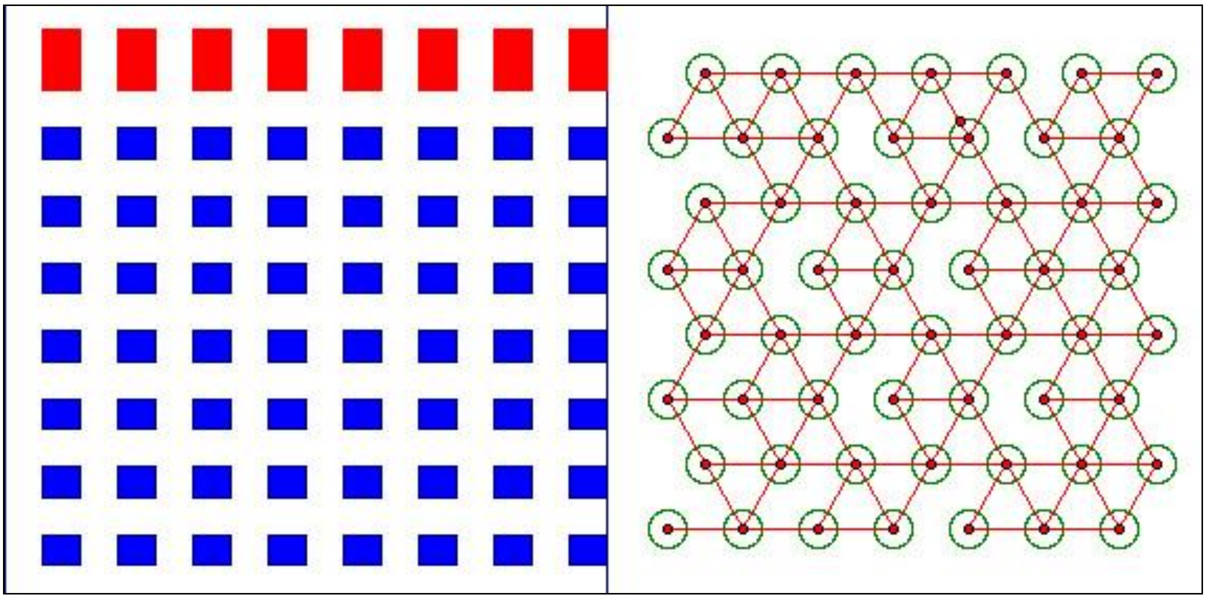


Figure 5. Part of a student's modeling solution to the Restaurant problem.

The graphic representation was constructed accurately. More importantly, the student was able to save time from the details of the layout and concentrate on analyzing the mathematical relationships embedded in the layout and real life considerations to achieve deeper, conceptual understanding. This can be seen by part of the student's analysis shown below:

After a Google search for the phrase "seats 10 to 12 people," I found the Web site of a catering company that listed a bunch of different table sizes and how many people can sit at each table (see attached). A 4-to-6 person rectangular table was 4 feet by 30 inches (2.5 feet). Thus, I figured that a standard 2-person table would be 2 feet by 2.5 feet. My first thought was that the most convenient setup would be to have just that one type of table: the 2-person 2' x 2.5' table. That way, two or more of these tables could be put together to seat parties of 4, 6, 8 or more. I figured that restaurant patrons would not want to be seated with people they didn't know. Some restaurants have mostly large tables and do this, but the vast majority tends to seat each party separately. I tried to make the model fit this requirement. If the restaurant were filled only with these tables for two, I asked myself which orientation would allow you to put more of the tables in the restaurant? If you positioned them lengthwise, i.e. the 2.5-foot side along the 80-foot wall, and the 2-foot side along the 40-foot wall, allowing for 2.5 feet of space between the wall and any table, as well as between any two tables, if x is the number of tables across, then 2.5x + 2.5(x+1) = 80, thus 5x + 2.5 = 80, then x rounds down to 15 tables across. If y is the number of tables vertically, then 2y + 2.5(y+1) = 40, thus 4.5y + 2.5 = 40, and y rounds down to 8 tables up and down, for 120 tables, seating 240 people. (If we did this in only half the restaurant, there would be 60 tables and 120 people.) If you did it with the tables facing the other way, you'd have 4.5x + 2.5 =80 and 5y + 2.5 = 40, and x and y would be 17 and 7, respectively, leading to 119 tables. So the other orientation is better.

It is not impossible to explore and model this problem situation without dynamic geometry software, but that would be much more inconvenient and time consuming. In addition, that results in less time spent on conceptual oriented tasks.

4. Creating Teaching Projects That Take Advantage of the Use of GSP

The technology course was designed for prospective teachers. Eventually, students enrolled in this course will apply what they have learned to their classroom teaching. Therefore, students should experience using GSP as both a learning tool and a teaching tool. For this reason, students were requested to create different projects for a high school geometry class in which the use of GSP was required or at least encouraged. The projects required children to construct their own sketches and to write a proof explaining/justifying their observations. Each project integrated previously learned material with new properties or ways of thinking about proof. The goals for the projects were:

- To guide children to learn more about using GSP as a learning tool through constructing their own sketches;
- To emphasize the versatility of the theorems and properties that children have studied by applying them to new situations;
- To empower children to draw conclusions based on observation and justify their statements with rigorous mathematical proof.

Assuming that the children had not yet taken on the tasks of both making new observations and proving them by themselves, the projects were required to have guiding questions to help to lead them through the process. The children were familiar with the GSP construction tools, but they were not experts. Therefore, in describing the steps for construction, the projects had to be very clear.

The guiding questions for the proof should be specific enough to point out important parts of the construction, yet open enough that the children would have to figure things out on their own, playing with the dynamic nature of the software and answering the questions for themselves. If the children got stuck, their teacher would prompt them to take different measurements, and then drag different corners of their shapes. Then the teacher would ask more guiding questions: What changed? What stayed the same? Why is that? What do the different shapes share or have in common?

One of the most powerful outcomes of the projects would be the integration and application of previously learned theorems and properties to new conjectures. Many projects met these requirements and were considered to be good projects by both the author and the whole class of students. The following project gives an example.

"A Twist on the Pythagorean Theorem"

We are familiar with Pythagorean Theorem and we have proven it in several ways. Let's think more about the idea of "squared" geometrically rather than algebraically. What if instead of computing the side squared we computed the side equilateral triangle-d or the side regular hexagon-ed or the side regular pentagon-ed. Can we construct other regular polygons (with any number n of sides) besides squares for which this equation will hold?

You will use the Geometer's Sketchpad to illustrate this question, and then you will prove your observation using the formula for the area of a regular polygon.

Follow these steps twice – with different values for n. Create these on two different pages within the same file. To do this, go to File -> Document Options and choose Add Page -> Blank Page. Name your pages by the name of the n-gon featured. If you can make it general enough, you only have to write one proof.

1. *Construct a right triangle using the "construct perpendicular" feature of the Geometer's Sketchpad.*
2. *Choose a number of sides n for a regular polygon. Use the formulas discussed in class to compute the angle measurement of each interior angle of your polygon.*
3. *One at a time, rotate the sides of your right triangle n times to create a regular n-gons on the three sides of your triangle*
4. *One at a time, select all of the vertices of an n-gon and find its area.*
5. *What is the relationship between these areas? Is it what you would expect, using the Pythagorean Theorem as a model? Use Measure -> Calculate to verify this relationship, and drag a vertex of the right triangle to verify that it is generally true.*

Your Proof:

What you have observed can be proven using the area formula for a regular polygon. To keep your work as general as possible, you will use variables instead of the numbers that you can find through measuring on Geometer's Sketchpad. Work out this proof on your own, and then type it into your Sketchpad file. In addition to this text, you should display the calculations done in steps 4 and 5 above.

The students who created this project also provided a GSP file giving a sample solution (Figure 6). This solution was not perfect, but the general idea was correct and clear.

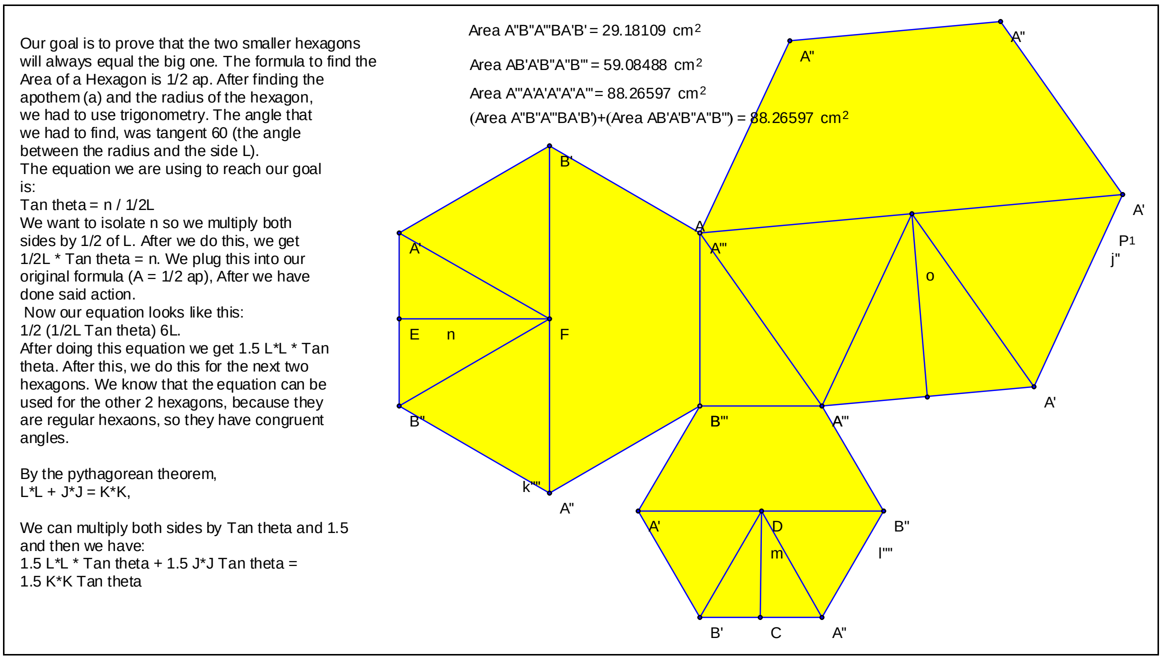


Figure 6. A GSP file along with the project *"A Twist on the Pythagorean Theorem"*

From this project, we can see that children were required to be engaged in the hands-on and minds-on exploration activities that took full advantage of the power of GSP. This student-centered project was fun, challenging, and different from the usual work out of the textbook. It was a powerful blend of discovery with integration and application of previous knowledge, which I believe is at the heart of what mathematics really is.

5. Conclusion

My experience in teaching the Technology course at NYU indicates that the preservice teachers have benefited from exploring mathematics and mathematics teaching with technology and especially GSP. The examples given above point out that for some of the challenging problems that are presented to students, it is almost impossible or very hard to manually make correct drawings. To overcome this difficulty, the use of dynamic geometry software seems to be critical, or at least very desirable. In addition, the use of the software can stimulate students' insight of problem solving and provide an easy and convincing way of verifying the solution. (Of course, the verification is only valid for specific numeric example, which cannot be generalized. The symbolic exploration using other software in addition to numerical software such as GSP is needed later.) Moreover, students can construct accurate visual representations to model real world situations very efficiently by using transformations in dynamic geometry software. This can save time significantly so that students can concentrate on more conceptual oriented tasks. Good teaching projects that take advantage of dynamic geometry software can also effectively enhance school children's mathematics learning.

I administered a pretest and a posttest at the beginning and the end of the course, using Choi-koh's (see [1]) instrument to assess the students' van Hiele levels of geometric thinking. An initial analysis provided evidence that almost all students enrolled raised their geometric thinking by at least one van Hiele level by the end of the course, with most of them having progressed to van Hiele level 3 (abstract/relational) or 4 (deduction). Further research is needed to assess the causal linkages between the use of GSP and these outcomes. As NCTM indicates in its Standards documents (see [9], [10], and [11]), the utilization of technology in the learning and teaching of mathematics is essential. The place of technology as tools for learning and teaching mathematics is rapidly increasing, and the diverse uses are multiplying. It is, therefore, important to institute broad use of technology into the mathematics education programs. A technology course is necessary and strongly encouraged to be included in any secondary mathematics teacher preparation program.

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Supplemental Electronic Materials

[a] Jiang, Z. *Geometer's Sketchpad file for the readers to do explorations, 2007.*
[b] Jiang, Z. *Geometer's Sketchpad file for the readers to do verifications, 2007.*
[c] Jiang, Z. JavaSketchpad for readers to do explorations.
[d] Jiang, Z. JavaSketchpad for readers to do verifications.